ABSTRACT: Second-Order Logic (SOL): Logic or "Set Theory in Sheep's Clothing"?

This talk will consider two related but distinct issues. The first issue is whether second-order logic (SOL)—and, by implication, higher-order logic, in general—really should be considered as a 'true logic', on all fours with familiar first-order logic (FOL), or whether SOL is actually set theory masquerading as logic. The second issue is the dispute between logical monists and logical pluralists. Is it the case, as the monists maintain, that there is but One True Logic (OTL), which alone correctly analyzes the relation of logical consequence (*logical* entailment) and the property of *logical* validity? Or, as the logical pluralists maintain, is there a plurality of legitimate logics, perhaps specifying a plurality of legitimate conceptions of logical consequence and logical validity? The second issue impinges on the first in a fairly obvious way: it would seem to be *prima facie* less likely that SOL might turn out to be the single OTL than that SOL might end up being endorsed as one of a plurality of legitimate logics, specifying one of a plurality of legitimate conceptions of logical consequence/logical validity.

After quickly and informally specifying the principal differences between SOL and FOL, I turn to the debate between logical monists and logical pluralists. I suggest that the attraction of monism derives from a venerable Rationalist assumption that there is a *single rational structure* of Thought and/or Reality, embedded in natural language, and the consequent assumption that this structure yields a single set of 'Laws of Thought', which specify a single conception of logical consequence/logical validity. A relaxation of this assumption yields what I call logical unitarianism: the idea that there is a nebula of Virtues/Vices of logics that can be used to construct a conception of 'logicality' in terms of which we may make honorific comparisons among different logics. I myself endorse a Carnapian or eclectic form of logical pluralism, according to which: (1) logical consequence/validity is always relative to its specification by a formal logic or class of formal logics; (2) there is no question of the 'legitimacy' of logics beyond the satisfaction of some constraints (e.g., non-triviality, coherence) on their formal construction; (3) logics can be legitimately compared and contrasted with respect to their abilities to serve *specific uses* or *applications*.

The final section of the talk addresses two issues which have been thought to give FOL an 'honorific edge' over SOL: (i) the fact that FOL is complete, while SOL is inherently incomplete and (ii) the relatively greater 'entanglement' of SOL with issues in set theory. I argue that the incompleteness of SOL has the same source as the undecidability of FOL and, further, that the *theoretical limitations* of axiomatizations of FOL and SOL need to be distinguished from (and are, in some ways, less important than) the *practical limitations* of those axiomatizations. I also argue that the very fact that set theory is used in the standard semantic specification of the logical-consequence relation for both FOL and SOL means that the camel of set-theoretic 'entanglement' already has its nose under the tent in the case of FOL: although there is an undeniable difference in *degree*, the *kind* of set-theoretic entanglement is the same for both FOL and SOL. In conclusion, I interpret Väänänen's claim of the essential equivalence between SOL and set theory formulated in FOL when employed as foundations of mathematics as suggesting that it is equally correct/incorrect (and equally inconsequential) to characterize SOL as *set theory in sheep's clothing* and to characterize first-order set theory as *logic in sheep's clothing*.

Michael J. White